Name___________________________

Students entering Calculus Summer packet

Each week this summer please complete the following review sheets. Please show as much work as you can for each problem. These review sheets will be collected on Monday, September 9 and will be counted as a quiz grade for the first quarter. The pages will be graded for accuracy and completion. Doing this review will help you to prepare for the PreCalculus/College algebra skills.

Have a great summer!

York Prep Math Department.

Hello Future Calculus Student,

This assignment must be completed and handed in on Mon., Sept. 9. This assignment will serve as the main review for a test on this material. The test will be administered during the first two weeks of classes.

You must complete the entire assignment showing sufficient evidence of effort in mathematical reasoning, use of computational skills, understanding of concepts, and communication of appropriate mathematical processes and terms. In other words, **show all your work or explain how you arrived at the solution, circle your answers, label when necessary, and answer each word problem in complete sentences, etc.**

You should only use pencil. No pens, markers, etc. allowed in notebook or binder. Calculators may be used and are recommended. You will be expected to have a TI-84 or the like. When using calculators, show work by writing any expression that you enter into the calculator.

If you have any questions, feel free to email spovshko@yorkprep.org.

Have a great summer!

Good luck and enjoy,

York Preparatory School Mathematics Department

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1, which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \circ \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \circ \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1) $\dfrac{\frac{25}{a}-a}{5+a}$

2) $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3) $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4) $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5) $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6) $f(2) =$ ____________ 7) $g(-3) =$ ____________ 8) $f(a+1) =$ ____________

9) $f\left[g(-2)\right] =$ ________ 10) $g\left[f(m+2)\right] =$ ________ 11) $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12) $f\left(\dfrac{\pi}{2}\right) =$ __________ 13) $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$**. Find each.**

14) $h\left[f(-2)\right] =$ ______ 15) $f\left[g(x-1)\right] =$ ______ 16) $g\left[h(x^3)\right] =$ ______

Find $\frac{f(x+h)-f(x)}{h}$ **for the given function** ***f*****.**

17) $f(x)=9x+3$

18) $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int.}\ (Let\ y=0)$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$y-\text{int.}\ (Let\ x=0)$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

19) $y=2x-5$

20) $y=x^2+x-2$

21) $y=x\sqrt{16-x^2}$

22) $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug x=3 *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$	$5^2 - y^2 - 9 = 0$
$-y^2 = 0$	$16 = y^2$
$y = 0$	$y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23) $x + y = 8$
$4x - y = 7$

24) $x^2 + y = 6$
$x + y = 4$

Interval Notation

25) Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

26) $2x - 1 \geq 0$

27) $-4 \leq 2x - 3 < 4$

28) $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

29) $f(x) = x^2 - 5$

30) $f(x) = -\sqrt{x+3}$

31) $f(x) = 3\sin x$

32) $f(x) = \frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

33) $f(x) = 2x + 1$

34) $f(x) = \dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9$$

$$= x-9+9$$

$$= x$$

$$g(f(x)) = \frac{(4x+9)-9}{4}$$

$$= \frac{4x+9-9}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

35) $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

36) $f(x) = 9 - x^2,\ x \geq 0$ $g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

37) Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

38) Determine the equation of a line passing through the point (5, -3) with an undefined slope.

39) Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

40) Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

41) Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

42) Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

43) Find the equation of a line passing through the points (-3, 6) and (1, 2).

44) Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\dfrac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\dfrac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

45) Convert to degrees: a. $\dfrac{5\pi}{6}$ b. $\dfrac{4\pi}{5}$ c. 2.63 radians

46) Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

47) Sketch the angle in standard position.

a. $\dfrac{11\pi}{6}$ b. 230° c. $-\dfrac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

48) Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

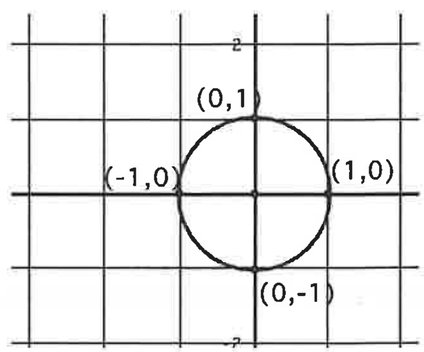
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

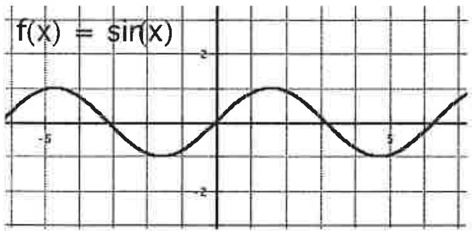
You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

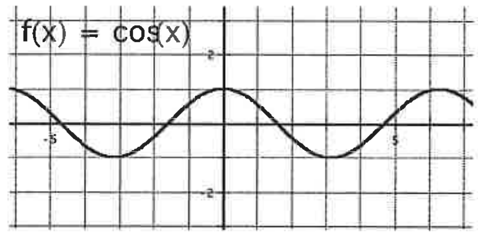
Example: $\sin 90^\circ = 1$ $\cos\frac{\pi}{2} = 0$



49) *a.*) $\sin 180^\circ$ *b.*) $\cos 270^\circ$ *c.*) $\sin(-90^\circ)$ *d.*) $\sin\pi$ *e.*) $\cos 360^\circ$ *f.*) $\cos(-\pi)$

Graphing Trig Functions





y = sin x and y = cos x have a period of 2 π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

50) $f(x) = 5\sin x$

51) $f(x) = \sin 2x$

52) $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

53) $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

54) $\sin x = -\frac{1}{2}$

55) $2\cos x = \sqrt{3}$

56) $\cos 2x = \frac{1}{\sqrt{2}}$

57) $\sin^2 x = \frac{1}{2}$

58) $\sin 2x = -\frac{\sqrt{3}}{2}$

59) $2\cos^2 x - 1 - \cos x = 0$

60) $4\cos^2 x - 3 = 0$

61) $\sin^2 x + \cos 2x - \cos x = 0$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

69) $f(x)=\frac{1}{x^2}$

70) $f(x)=\frac{x^2}{x^2-4}$

71) $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

72) $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

73) $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

74) $f(x)=\frac{4x^5}{x^2-7}$

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$ $= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Power property: $\log_b m^p = p \log_b m$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0